

# Transparent yttria produced by spark plasma sintering at moderate temperature and pressure profiles

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## Abstract

Transparent yttria ( $Y_2O_3$ ) bodies were fabricated by spark plasma sintering, and the effects of the sintering temperature on relative density, microstructure, and the optical and mechanical properties of  $Y_2O_3$  bodies were investigated. Fully dense  $Y_2O_3$  bodies were obtained at sintering temperatures 1473–1873 K. The average grain size was 0.24–0.32  $\mu\text{m}$  at 1473–1573 K, and steadily increased to 1.97  $\mu\text{m}$  with an increase in temperature to 1823 K. The highest transmittance was obtained in the  $Y_2O_3$  body sintered at 1573 K and annealed at 1323 K, showing 81.7% (99% of the theoretical value) at a wavelength of 2000 nm.

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**Keywords:**  $Y_2O_3$ ; Sintering; Optical property; Grain growth; Spark plasma sintering

## 1. Introduction

Yttria ( $Y_2O_3$ ) is a promising material for application in high-power lasers,<sup>1</sup> infrared windows<sup>2</sup> and high-temperature refractories.<sup>3</sup> Since an  $Y_2O_3$  single crystal is difficult to grow owing to its high melting point (2700 K),<sup>4</sup> transparent polycrystalline ceramic obtained by sintering is an alternative for practical applications. Transparent  $Y_2O_3$  ceramic has been fabricated by pressureless sintering in a vacuum<sup>5–7</sup> or in a  $H_2$  atmosphere,<sup>8</sup> hot pressing<sup>9,10</sup> and hot isostatic pressing.<sup>11,12</sup> In these methods, however, the sintering temperatures are high (above 1853 K) and the grain size is large, usually several ten of micrometers in diameter. Large-grained ceramics tend to have low strength, which deteriorates further in practical applications.

$Y_2O_3$  bodies with a submicrometer grain size have recently been prepared by the hot isostatically pressed two-step sintering method.<sup>13</sup> Spark plasma sintering (SPS) is another candidate for fabrication of transparent ceramics with submicron grains. The distinctive features of the SPS process are fast densification, short holding time and low sintering temperature.<sup>14</sup> An  $Y_2O_3$

body with a relative density of 97% has been prepared by SPS at a low sintering temperature of 1123 K with a heating rate of 0.17  $\text{K s}^{-1}$  for 3.6 ks.<sup>15</sup> Chaim et al. reported an  $Y_2O_3$  body with a relative density greater than 98% prepared by SPS at 1673 K and 3  $\text{K s}^{-1}$  for 300 s.<sup>16</sup> However, these  $Y_2O_3$  bodies were not transparent because their relative densities were not high enough; it should be greater than 99%. By using either a significantly low heating rate (0.03  $\text{K s}^{-1}$ )<sup>17</sup> or a high pressure (300 MPa),<sup>18</sup> transparent  $Y_2O_3$  bodies have been obtained by SPS.

In this study, we prepared a highly transparent  $Y_2O_3$  body at moderate temperature and pressure profiles, and investigated the effects of SPS sintering conditions on the microstructure, and the optical and mechanical properties of  $Y_2O_3$  bodies.

## 2. Experimental procedure

$Y_2O_3$  powder (purity: 99.999%, Jiahua Corp., China) was used as the starting material. The as-received powder was ball milled in ethanol using zirconia balls for 12 h and dried at 333 K in an oven for 24 h. The powder was ground and sieved through a 200-mesh grid and calcined at 1273 K in air for 7.2 ks in an electric furnace.

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The calcined powder was poured into a graphite die with an inner diameter of 10 mm and sintered by SPS (SPS-210 LX, Fuji Electronic Industrial Co., Ltd, former SPS Syntex Inc., Japan) in a vacuum. The temperature of the graphite die surface was measured using a radiation pyrometer. The sintering temperature was increased to 873 K in 180 s and to 1373 K in 300 s, and then held for 300 s. The sintering temperature was further increased to 1373–1823 K at a heating rate of  $0.17 \text{ K s}^{-1}$  and maintained at that temperature for 2.7 ks. A pressure of 10 MPa was pre-loaded between room temperature and 1373 K and a final pressure of 100 MPa was loaded above 1373 K and maintained until the end of the sintering process. Post-annealing was carried out at 1123–1423 K for 21.6 ks in air. Both sides of the sintered bodies were mirror-polished using a diamond slurry ( $1 \mu\text{m}$ ).

The specific surface area of the  $\text{Y}_2\text{O}_3$  powder after calcination was measured by the Brunauer–Emmett–Teller (BET) method using a surface area and pore size analyzer (TriStar 3000, Micromeritics Instruments Corp., USA) with nitrogen ( $\text{N}_2$ ) adsorption at room temperature. The particle size ( $d_{\text{BET}}$ ) was calculated using the following equation:

$$d_{\text{BET}} = \frac{6}{\rho \times S_{\text{BET}}}, \quad (1)$$

where  $\rho$  is the theoretical density of  $\text{Y}_2\text{O}_3$  ( $5.031 \text{ Mg m}^{-3}$ ) and  $S_{\text{BET}}$  is the specific surface area measured by the BET method. The density of the sintered bodies was measured by the Archimedes method in distilled water. The crystal phase was identified by X-ray diffraction (XRD, RAD-2C, Rigaku Corp., Japan) using graphite monochromatic  $\text{CuK}\alpha$  radiation (wavelength:  $0.154 \text{ nm}$ ). The polished surfaces of the  $\text{Y}_2\text{O}_3$  bodies were thermally etched in air for 3.6 ks at 100–200 K below the sintering temperature. The morphology of the  $\text{Y}_2\text{O}_3$  powders and the microstructure of the sintered bodies were observed by a field emission scanning electron microscope (FESEM, JSM-7500F, JEOL Ltd., Japan) and a scanning electron microscope (SEM, S-3100H, Hitachi Ltd., Japan). The average grain size was determined from the linear intercept length of the thermally etched surfaces using FESEM and SEM micrographs (assuming a grain size 1.56 times the mean intercept) with at least 250 grains counted.<sup>19</sup> In-line transmittance was measured using a spectrophotometer (UV-3101PC, Shimadzu Corp., Japan) in the wavelength between 190 and 2500 nm. The thickness of the specimens was about 1 mm. Vickers hardness ( $H_V$ ) was measured by a hardness tester (HM-221, Mitutoyo Corp., Japan) at a load ( $P$ ) of 2.94 N at room temperature. The fracture toughness was calculated by Eq. (1) using the half length of the crack ( $c$ ) formed around the corners of the indentations:<sup>20</sup>

$$K_{\text{IC}} = 0.073 \times \frac{P}{c^{1.5}}. \quad (2)$$

### 3. Results and discussion

Fig. 1 shows an FESEM image of calcined  $\text{Y}_2\text{O}_3$  powder. The powder was slightly agglomerated with nearly spherical grains

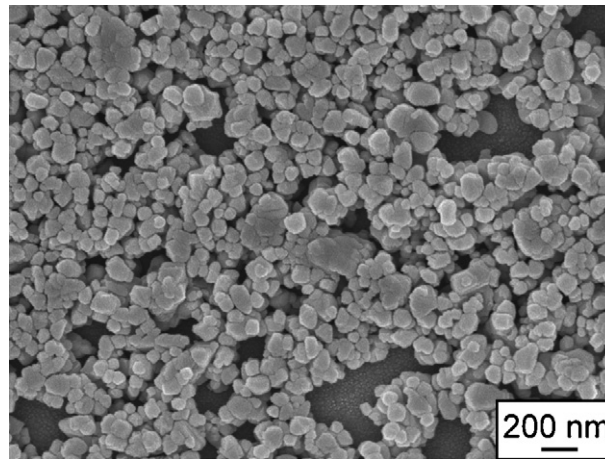


Fig. 1. FESEM image of  $\text{Y}_2\text{O}_3$  calcined powder.

having an average size of about 100 nm. The specific surface area (BET) of the calcined powder was  $12.8 \text{ m}^2 \text{ g}^{-1}$ , and the calculated equivalent particle size was 93 nm, which was well in accordance with the result from the FESEM.

Fig. 2 shows XRD patterns of calcined  $\text{Y}_2\text{O}_3$  powder and  $\text{Y}_2\text{O}_3$  bodies sintered at 1573 K before and after annealing. The patterns were indexed as cubic  $\text{Y}_2\text{O}_3$  (space group:  $Ia\bar{3}$ ;  $a = 1.06041 \text{ nm}$ ; JCPDS no. 41-1105) in all the specimens. Sharp peaks were observed in the sintered  $\text{Y}_2\text{O}_3$  bodies, *i.e.*, the (2 2 2) peak had a full-width at half-maximum of  $0.471^\circ$  and  $0.188^\circ$  before (Fig. 2a) and after sintering (Fig. 2b and c), respectively. The lattice parameters of  $\text{Y}_2\text{O}_3$  bodies sintered at 1573 K before (Fig. 2b) and after (Fig. 2c) annealing were 1.06048 (0.00005) and 1.06057 (0.00006) nm, respectively. These were in agreement to the reported value (1.06041 nm).

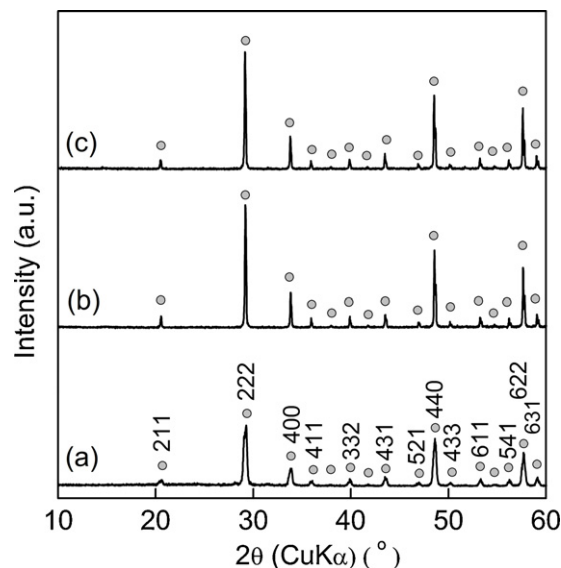


Fig. 2. X-ray diffraction patterns of (a)  $\text{Y}_2\text{O}_3$  calcined powder, and  $\text{Y}_2\text{O}_3$  bodies sintered at 1573 K (b) and (c) after annealing at 1323 K for 21.6 ks in air.

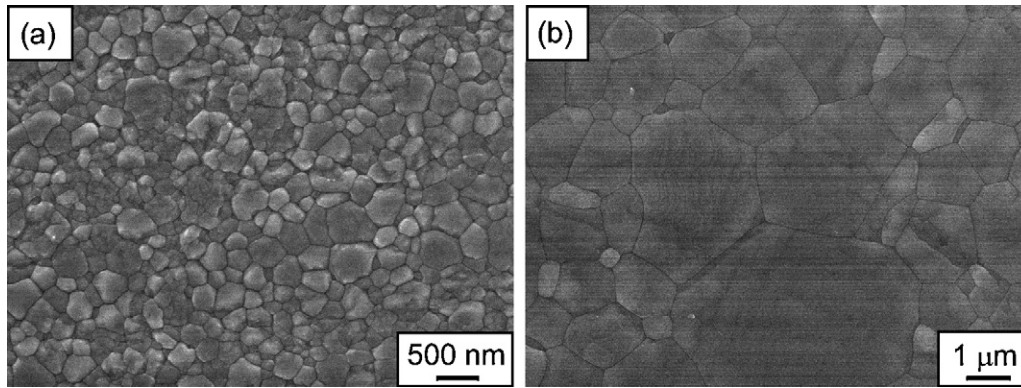


Fig. 3. FESEM images of  $\text{Y}_2\text{O}_3$  bodies sintered at (a) 1573 K and (b) 1823 K.

Fig. 3 shows FESEM images of  $\text{Y}_2\text{O}_3$  bodies sintered at 1573 and 1823 K. Pores were rarely observed in either of the specimens. The grains were uniform with an average grain size of  $0.32 \mu\text{m}$  at a sintering temperature of 1573 K (Fig. 3a), whereas they grew to  $1.97 \mu\text{m}$  and the grain size distribution was widened at 1823 K (Fig. 3b). Fig. 4 shows the effect of sintering temperature on the relative density and average grain size of  $\text{Y}_2\text{O}_3$  bodies sintered at 1373–1823 K. The relative density was 98% of the theoretical value at 1373 K and fully dense bodies (above 99%) were obtained in the temperatures 1473–1823 K. The average grain size slightly increased from  $0.24$  to  $0.32 \mu\text{m}$  at sintering temperatures 1473–1573 K, and significantly increased to  $1.97 \mu\text{m}$  with an increase in temperature to 1823 K.

Fig. 5 shows FESEM images of the fracture surfaces of  $\text{Y}_2\text{O}_3$  bodies sintered at 1473–1773 K. The fracture mode was mainly transgranular. Small pores were observed at the triple junction of the grain boundaries and grain interiors in all the specimens (as indicated by open circles in Fig. 5).

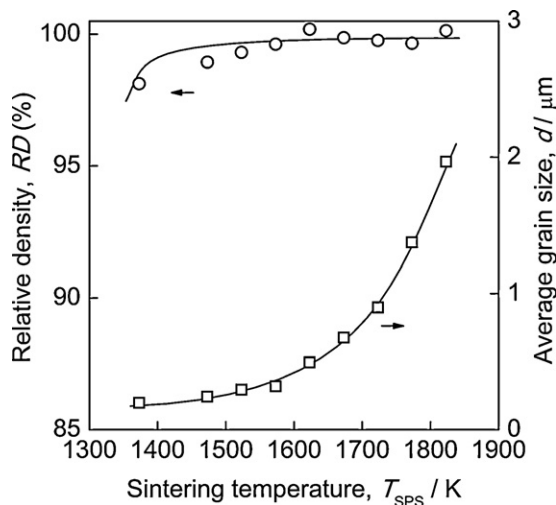


Fig. 4. Effect of sintering temperature on relative density and average grain size of  $\text{Y}_2\text{O}_3$  bodies sintered by SPS.

Fig. 6 shows photographs of  $\text{Y}_2\text{O}_3$  bodies sintered at 1573–1723 K before and after annealing at 1323 K for 21.6 ks in air. The text 30 mm underneath the  $\text{Y}_2\text{O}_3$  bodies can be seen and all the specimens were dark gray (Fig. 6a–c). The gray color of the  $\text{Y}_2\text{O}_3$  body sintered at 1573 K disappeared after annealing (Fig. 6d), while that of the  $\text{Y}_2\text{O}_3$  bodies sintered at 1673 and 1723 (Fig. 6e and f) partially disappeared.

Fig. 7 shows transmittance spectra of  $\text{Y}_2\text{O}_3$  bodies sintered at 1573 K before and after annealing at 1123–1423 K for 21.6 ks. With an increase in annealing temperature, the gray color disappeared gradually and transmittance increased over the entire wavelength range. The optimal annealing temperature was 1323 K (Fig. 7b). The ultraviolet absorption edge was blue-shifted from 308 to 256 nm with increasing annealing temperature from 1123 to 1223 K, and there was almost no change at higher annealing temperatures up to 1423 K.

Fig. 8 shows transmittance spectra of  $\text{Y}_2\text{O}_3$  bodies sintered at 1473–1773 K after annealing at 1323 K for 21.6 ks in air (solid lines) and those calculated from the refractive index of an  $\text{Y}_2\text{O}_3$  single crystal (dashed line). In-line transmittance ( $T$ ) can be calculated by the Beer–Lambert law using the following equation<sup>21</sup>:

$$T = \frac{2n}{1+n^2} e^{-\alpha_t d}, \quad (3)$$

where  $n$  is the refractive index,  $\alpha_t$  is the total absorption and  $d$  is the thickness of the specimen. Theoretical transmittance is depicted in Fig. 8 (dashed line) by substituting  $n$  of an  $\text{Y}_2\text{O}_3$  single crystal<sup>22</sup> assuming  $\alpha_t = 0$  in Eq. (3).  $n$  of the  $\text{Y}_2\text{O}_3$  single crystal was 1.89 at  $\lambda = 2000 \text{ nm}$ , where  $T$  was calculated as 82.6%. This value can be 100% of theoretical transmittance at  $\lambda = 2000 \text{ nm}$ . The transmittance of the  $\text{Y}_2\text{O}_3$  body increased significantly over the entire wavelength range after annealing. The  $\text{Y}_2\text{O}_3$  body sintered at 1573 K showed the highest transmittance of 55.0% and 81.7% at  $\lambda = 550$  and  $2000 \text{ nm}$ , respectively. The transmittance of the  $\text{Y}_2\text{O}_3$  body sintered at 1573 K after annealing approached 99% of the calculated value in the infrared range (Fig. 8a). The sintering temperature used in this study (1573 K) for preparing highly transparent  $\text{Y}_2\text{O}_3$  was significantly lower than that for pressureless sintering (above 1973 K),<sup>5–8</sup> hot



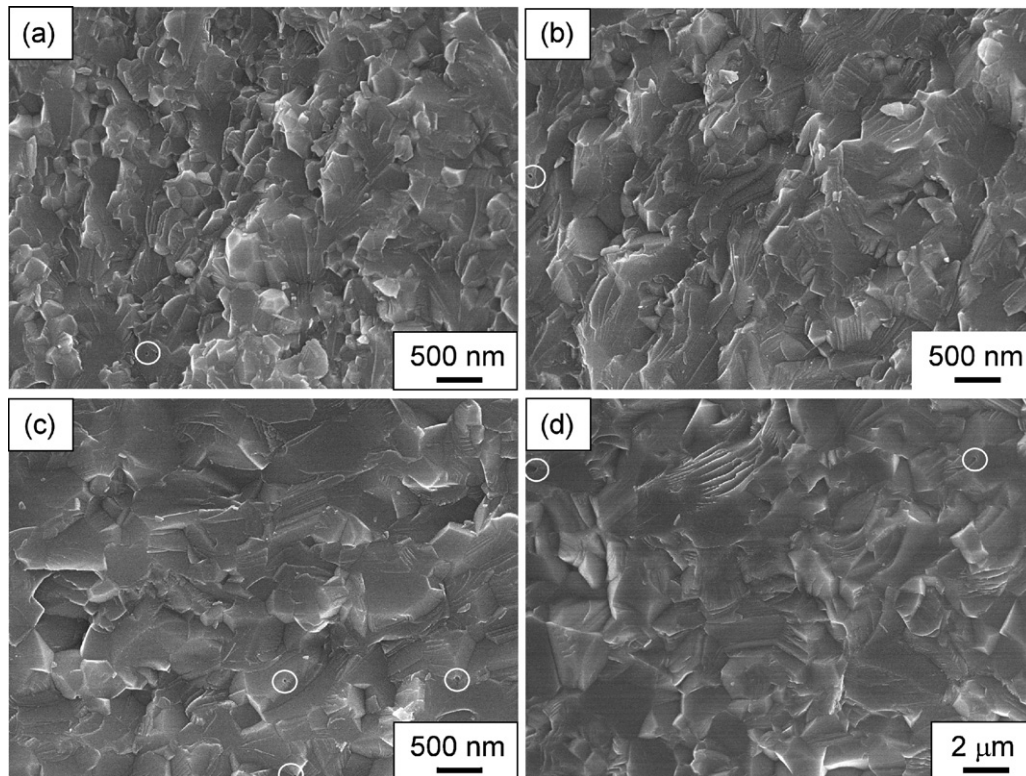


Fig. 5. FESEM images of fracture surfaces of  $\text{Y}_2\text{O}_3$  bodies sintered at (a) 1473 K, (b) 1573 K, (c) 1673 K and (d) 1773 K. Open circles indicate pores.

pressing (1853 K)<sup>10</sup> and hot isostatically pressed two-step sintering (1773 K).<sup>13</sup> The highest transmittance at  $\lambda = 700$  nm of the  $\text{Y}_2\text{O}_3$  body in this study ( $T = 68\%$ ) was higher than that of the  $\text{Y}_2\text{O}_3$  body prepared by SPS using a low heating rate of  $0.03 \text{ K s}^{-1}$  ( $T = 38\%$ ),<sup>17</sup> and was comparable to that of the  $\text{Y}_2\text{O}_3$  body prepared using a high pressure of 300 MPa ( $T = 68\%$ ).<sup>18</sup> It is known that an oxide vacancy readily forms in a vacuum during SPS, resulting in a gray color and degradation of transparency.<sup>23</sup> After annealing, the oxide vacancy can be eliminated, and thus the ultraviolet absorption edge was blue-shifted and the specimen became colorless.<sup>24,25</sup>

Fig. 9 shows the effect of sintering temperature on Vickers hardness ( $H_V$ ) and the fracture toughness of the annealed  $\text{Y}_2\text{O}_3$  bodies (Fig. 9a) and the relationship between  $H_V$  and grain size (Fig. 9b).  $H_V$  of the  $\text{Y}_2\text{O}_3$  body sintered at 1373 K was 8.3 GPa and had a maximum value of 9.0 GPa at 1523 K, the value decreasing to 7.6 GPa with an increase in sintering temperature to 1823 K. The fracture toughness ranged from 1.5 to  $1.0 \text{ MPa m}^{1/2}$  irrespective of sintering temperature, which was in accordance with the reported value.<sup>13,26</sup>  $H_V$  of the  $\text{Y}_2\text{O}_3$  body sintered at 1523 K with an average grain size of about  $0.29 \mu\text{m}$  was close to that of the  $\text{Y}_2\text{O}_3$  body subjected to hot isostatic pressing with almost the same grain size.<sup>13</sup> Fig. 9b shows  $H_V$  as a function of the inverse square root of the grain size ( $d$ ).  $H_V$  had a linear dependency with  $d^{-1/2}$ , obeying the Hall–Petch relation. Since the  $\text{Y}_2\text{O}_3$  body sintered at 1373 K had a low density of 98%, it did not obey the relation. It is commonly observed that  $H_V$  increases with  $d^{-1/2}$  in many ceramics, such as  $\text{MgO}$ ,<sup>27</sup>  $\text{MgAl}_2\text{O}_4$ ,<sup>28</sup> hydroxylapatite ( $\text{Ca}_{10}(\text{PO}_4)_6(\text{OH})_2$ )<sup>29</sup>

and  $\text{Al}_2\text{O}_3$ .<sup>30,31</sup> This can be attributed to the reducing free path for dislocation as the grain size decreases.<sup>30</sup> The  $\text{Y}_2\text{O}_3$  bodies in the current study, with grains having diameters  $0.24\text{--}1.97 \mu\text{m}$ , also followed this trend.

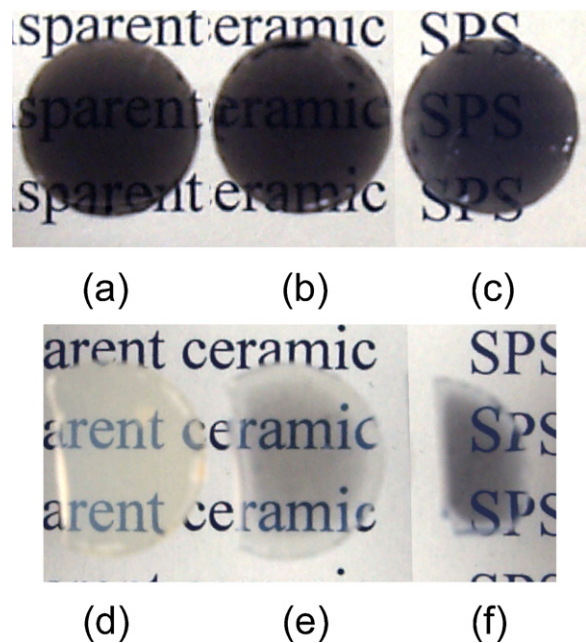


Fig. 6. Photograph of transparent  $\text{Y}_2\text{O}_3$  bodies sintered at (a) 1573 K, (b) 1673 K and (c) 1723 K before annealing, and sintered at (d) 1573 K, (e) 1673 K and (f) 1723 K after annealing at 1323 K for 21.6 ks in air. The text is 30 mm underneath the specimens.

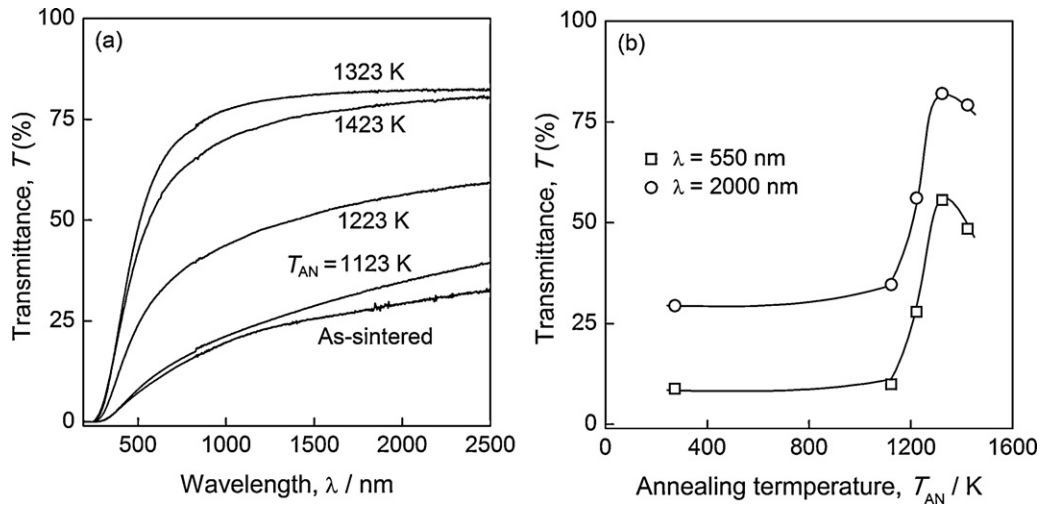


Fig. 7. (a) Transmittance spectra of  $Y_2O_3$  bodies before and after annealing at 1123–1423 K for 21.6 ks using specimens sintered at 1573 K, and (b) transmittance at  $\lambda = 550$  and 2000 nm plotted as a function of annealing temperature.

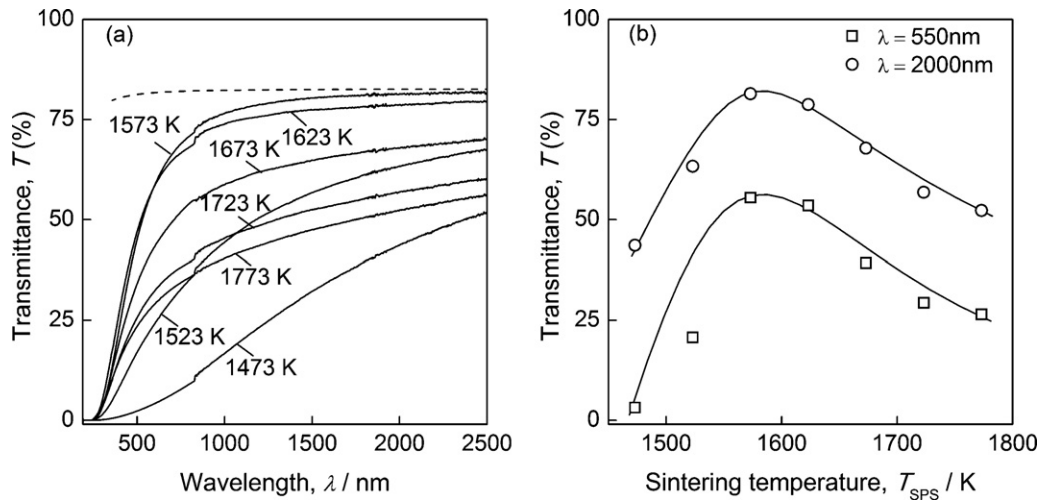


Fig. 8. (a) Transmittance spectra of  $Y_2O_3$  bodies sintered at 1473–1773 K after annealing at 1323 K, and (b) transmittance at  $\lambda = 550$  and 2000 nm plotted as a function of sintering temperature. The dashed line indicates transmittance calculated from the refractive index of  $Y_2O_3$  single crystal.<sup>22</sup>

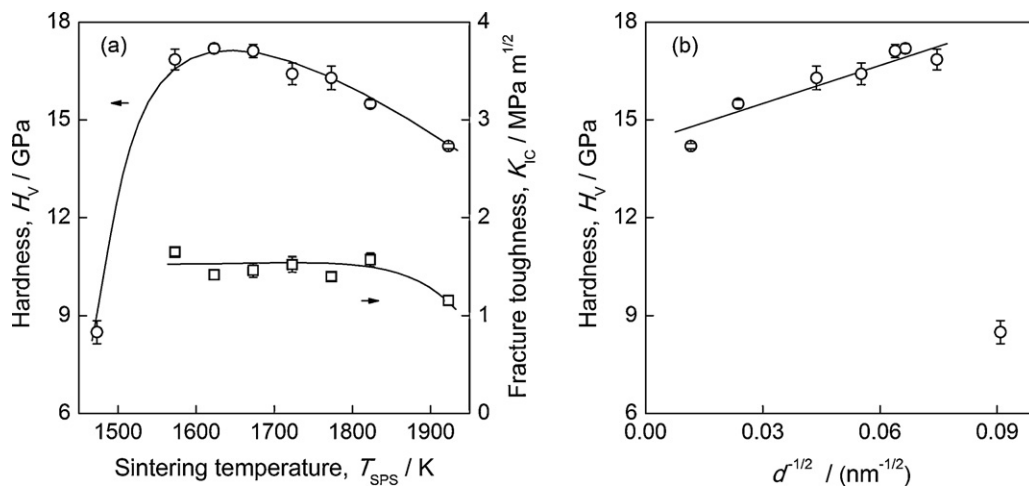


Fig. 9. (a) Effect of sintering temperature on Vickers hardness and fracture toughness of  $Y_2O_3$  bodies annealed at 1323 K, and (b) Vickers hardness as a function of the inverse square root of the grain size ( $d$ ).

#### 4. Conclusions

Transparent  $Y_2O_3$  bodies were fabricated by SPS at a low sintering temperature of 1573 K for 2.7 ks and annealing at 1323 K for 21.6 ks. Fully dense  $Y_2O_3$  was obtained at the sintering temperatures 1473–1823 K. The average grain size was 0.24–0.32  $\mu\text{m}$  at 1473–1573 K and steadily increased to 1.97  $\mu\text{m}$  with an increase in temperature to 1823 K. After annealing at 1323 K, the gray color of the  $Y_2O_3$  body sintered at 1573 K disappeared and the  $Y_2O_3$  body showed the highest transmittance of 81.6% at 2000 nm (99% of a theoretical value). The  $Y_2O_3$  body sintered at 1523 K had the highest hardness of 9.0 GPa.

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